Sum-fit method of analysis of nuclear decay spectra affected by extending dead-time

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We have developed a method of analysis of nuclear decay spectra affected by dead-time, in which a non-trivial, but relatively simple correction for dead-time losses is applied to produce an estimate of the spectrum in the absence of dead-time, with negligible bias and negligible deviation from the original Poissonian statistics. The method requires the measurement of arrival times of all observed decay events, but it does not require any knowledge about the dead-time or the nuclear decay. The parameters of nuclear decay can be determined in a subsequent straight-forward analysis of the estimated dead-time-free spectra, individually or in combinations. In a typical realistic situation this approach is expected to contribute 0.001% or less to the systematic error of nuclear half-life or activity. Compared to the exact method of analysis that we developed earlier [1], the present method is much easier to implement and produces the results significantly faster.

The key element in the analysis is the imposition of a known, sufficiently large extending dead-time to the observed sequence of events in order to produce a set of survived events for which the effects of the actual dead-time are completely obliterated by the effects of the imposed dead-time. As a result, the dead-time following each survived event and the live time preceding it are known exactly. The next step involves sorting the survived events into decay spectra, using a chosen channel width Δt.

For an infinitely long sequence of Poissonian events that starts at a known time and under known circumstances, a correction for the effects of an imposed dead-time may depend only on the nature of the dead-time and the product of the ideal rate ρ of the events (*i.e.,* the event rate in the absence of dead-time) and the imposed dead-time per event τ. However, if this sequence of events is limited to time (channel width) Δt, a correction for dead-time losses based exclusively on the information contained within the channel width may also depend on the dimensionless scaling parameter $\tau/\Delta t$.

Since the mathematical complexities associated with the effect of extending dead-time on time-binned Poissonian events are overwhelming [2], we chose to study the effects of dead-time correction using simulated events, with $\rho\tau$ ranging from 10^{-6} to 0.05 and $\tau/\Delta t$ ranging from 10^{-8} to 0.002. We found that a good estimate n_o of the number of original Poissonian events in each channel of a nuclear decay spectrum (that would be observed in the absence of dead-time) can be expressed as

$$n_o = \frac{\Delta t}{\tau}\ln\left(\frac{\Delta t}{t_{\text{live}}}\right) \left[1 + a\,\tau/\Delta t + b\ln\left(\frac{\Delta t}{t_{\text{live}}}\right) + c\,\frac{\tau}{\Delta t}\ln\left(\frac{\Delta t}{t_{\text{live}}}\right)\right]^{-1} . \qquad (1)$$

where t_{live} is the live-time within the channel. The recommended values of parameters a, b, and c were found to be those given in Table 1. Eq.(1) shows that the live-time fractions $t_{\text{live}}/\Delta t$ for each channel are the ultimate channel-specific parameters that define the decay spectra. The actual numbers of survived events in each channel are not needed.

To demonstrate the effectiveness of Eq.(1), we applied it to the most critical of the cases considered: the one with the largest values of $\tau/\Delta t$ (*i.e.*, 0.002) and $\rho\tau$ (*i.e.*, 0.05). Specifically, for an ideal

Table I. Recommended values of the parameters in Eq.(1). The parentheses enclose the umcertainty of the last significant figure.

Parameter	Value
a	0.5004(1)
b	$1.1(1)\cdot 10^{-4}$
c	0.17(1)

rate of 10 s^{-1}, this would imply τ = 5 ms and Δt = 2.5 s. The resulting distributions of the numbers of events per channel are shown in Fig.1.

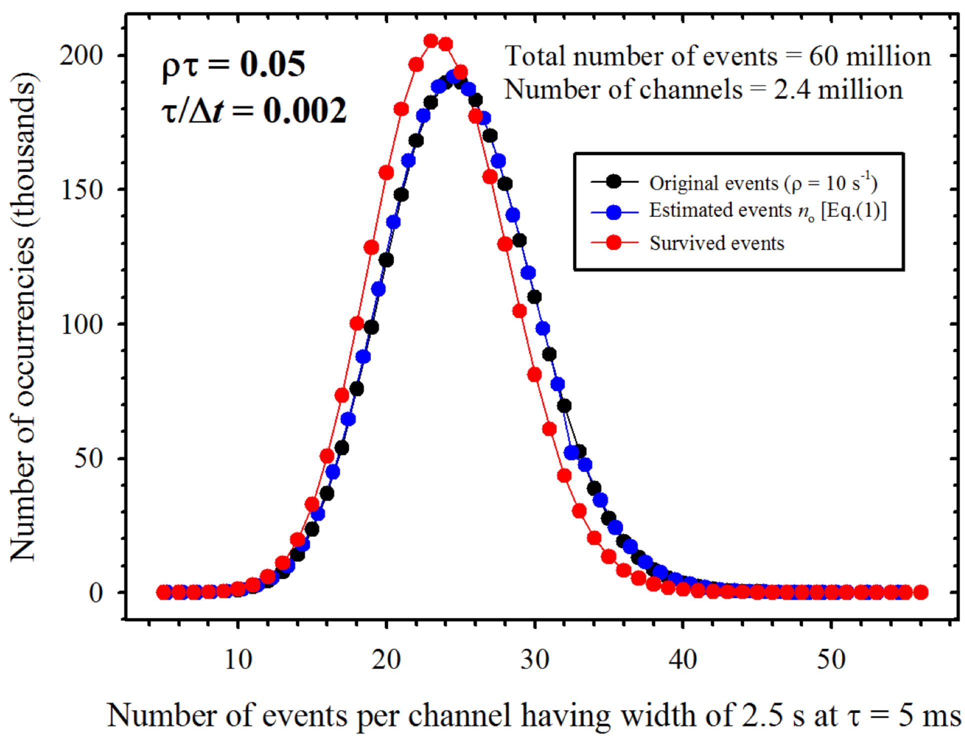

FIG. 1. Distribution of the actual number of events per channel (black); the number of events that survived after the dead-time was imposed (red); and the number of events deduced using Eq.(1) from the known channel live-time fractions (blue). Note that the ordinate of each blue data point corresponds to the number of occurrences of non-integer values that have the same integer part, while its corresponding abscissa is the (non-integer) average of those values. The small dip at an abscissa of about 33 is a consequence of the fact that the corrected numbers of counts are non-integers (shifted up from their integer parts) and are likely to be clustered around a relatively narrow range of typical values. A dip may be expected as this distribution starts to shift and/or broaden enough to extend over the next integer value (or further on). Error bars of the data points cannot be seen because they are smaller than the symbols used. The lines connecting the data points are there only to guide the eye.

Evidently, the distribution of the number of events deduced from the known channel live-time fractions (shown in blue) matches very well the Poissonian distribution of the actual numbers of events per channel (shown in black) before the dead-time was imposed. For the case shown, the former distribution has its mean within 0.02% of the expected value of 25 (which is within the range expected for a 60-million-event statistics), and the ratio of its actual variance to the expected mean value is within only 1.7% of unity, which is not expected to significantly affect the results in the analysis of the dead-time-corrected spectra. Because the number of events lost due to dead-time is larger when the original number of events is larger, the distribution of the actual number of survived events (shown in red) is narrower than the other two distributions and its mean is lower.

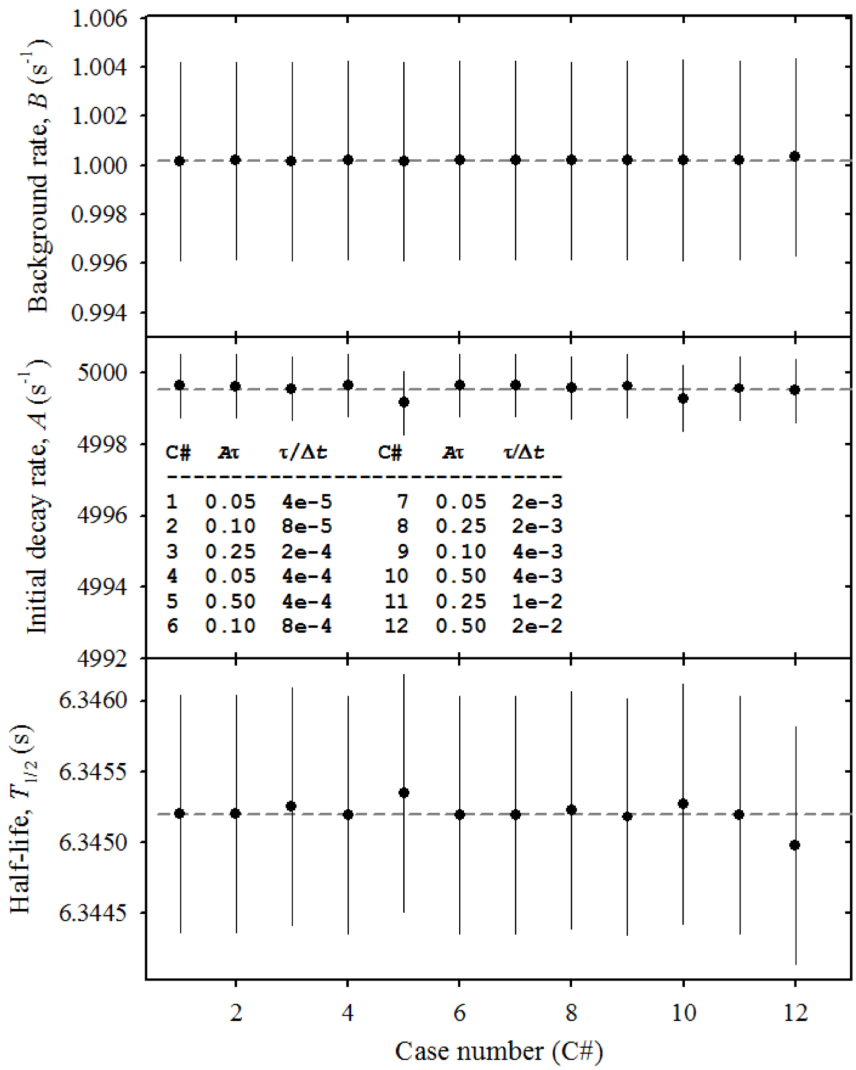

FIG. 2. Effects of using Eq.(1) for dead-time correction of the decay spectra on the results of a measurement of ^{26m}Al half-life. The events used were simulated, using $T_{1/2}$ = 6.3452 s for the half-life of ^{26m}Al, an assumed constant background event-rate of B = 1 s^{-1}, and an initial true decay rate of A = 5000 s^{-1}. The imposed extending dead-time per event τ was 10 μs , 20 μs, 50 μs, and 100 μs, while the channel width Δt was 5 ms, 25 ms, and 250 ms. All combinations of τ and Δt were used and numbered according to the table shown above. There were 1320 samples with duration of 125 s each, containing a total of about 60 million events. The dashed lines are there only to guide the eye.

The final demonstration of the effectiveness of Eq.(1) along with the values of *a*, *b*, and *c* from Table 1 is given in Fig. 2, which shows the effects of using Eq.(1) for dead-time correction of the decay spectra on the results of a typical measurement of nuclear half-life, such as that of ^{26m}Al. The events used were simulated. The same dead-time-free set of events was used in all the cases. Evidently, the use of Eq.(1) for dead-time correction of the decay spectra reproduces the expected results consistently and accurately within the region of its proven validity and somewhat beyond.

[1] V. Horvat and J.C. Hardy, Nuclear Instrum. Methods Phys. Res. **A713,** 19 (2013).
[2] W. Muller, Nucl. Instrum. Methods **117,** 401 (1974).